

the coupling imbalance is less than 1 dB. The isolation of the coupler is better than 30 dB over the frequency range measured by a digital voltmeter.

IV. CONCLUSION

An ultra-broad-band 3 dB coupler can be designed by taking advantage of strong coupling and dielectric waveguides with different dimensions. The bandwidth is much broader than that attainable in [1]–[8] while maintaining high directivity.

No extensive optimization techniques were used in the design of these couplers, and the dimensions and materials were chosen on the basis of convenience and availability.

Experimental results were presented in the frequency range 76–110 GHz which indicate agreement with the theoretical calculations (70–116 GHz). This ultra-broad-band coupler is simple to fabricate and has potential applications for millimeter, submillimeter, and optical wave bands.

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De-embedding and Unterminating Microwave Fixtures with Nonlinear Least Squares

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Abstract—This paper investigates a general method of characterizing microwave test fixtures for the purpose of determining the S parameters of devices embedded in the fixture. The accuracy of the technique was studied and compared with that of the common open–short–load and thru–reflect–line methods. Increased accuracy was obtained using redundant measurements.

I. INTRODUCTION

At microwave frequencies it is often impossible to directly measure the scattering parameters (S parameters) of a device under test such as a transistor or a microstrip circuit. Instead,

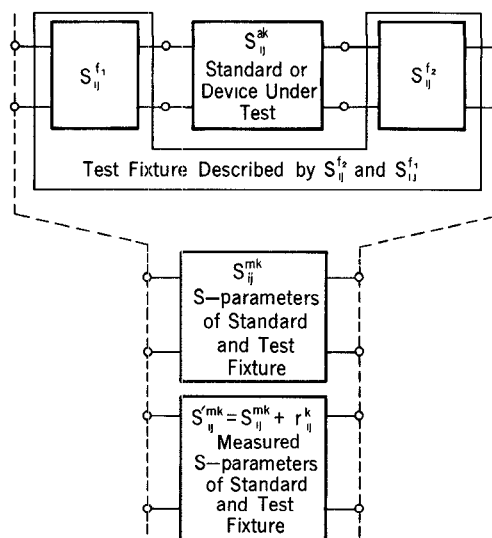


Fig. 1. The measurement configuration under consideration in this work. The S_{ij}^{mk} are the S parameters of the test fixture and inserted standard calculated from the ideal S parameters of the standard and the assumed S parameters describing the fixture. The S_{ij}^{mk} are the measured S parameters of the test fixture and the inserted standard.

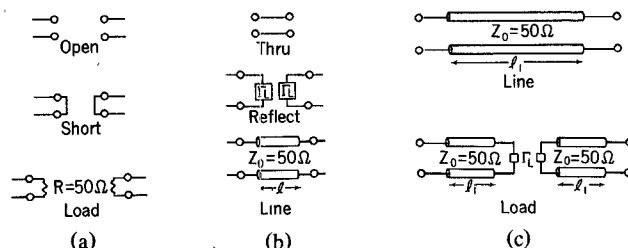


Fig. 2. The OSL, TRL, and more general standard sets investigated in this work are depicted schematically. (a) OSL standard set. (b) TRL standard Γ_L and γ unknown. (c) General standard set; Γ_L and γ may be unknown; Γ_i are known.

measurements are made at the reference plane of a network analyzer, which is removed both physically and electrically from the device under test by an intervening test fixture. Usually this fixture has high isolation between its input and output ports and can be described electrically by two two-port networks, as is illustrated in Fig. 1.

Once the S parameters of the two-port networks which describe the fixture are known, the S parameters of the embedded device may be determined from measurements at the reference plane of the analyzer. This procedure is straightforward and is referred to as de-embedding.

Generally it is not possible to directly measure the electrical characteristics of the fixture. The fixture must be characterized from measurements made at the analyzer reference plane when known devices, which we will call standards, are embedded in it. This process is referred to as unterminating [1].

An open, a short, and a matched load are used as the standards in the common open–short–load (OSL) technique of unterminating. These standards are depicted in Fig. 2(a). Each standard is connected in turn to each of the ports inside the fixture, resulting in six measured reflection coefficients. Under the assumption that the fixture is a reciprocal network, the

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numbers of measurements and unknowns are equal and the fixture may be unterminated.¹

Bauer and Penfield [1] have extended the OSL technique to include an arbitrary number of known loads. Using a least-squares algorithm, they showed that improved accuracy over the OSL technique can be obtained.

Franzen and Speciale [2] introduced the thru-reflect-line (TRL) method of unterminating. A thru line of minimum length, a thru line of a longer length, and a highly reflective load are used as standards; these are depicted in Fig. 2(b). Engen and Hoer [3] showed that because the number of measurements is larger than the number of unknowns, both the electrical delay of the thru line and the reflection coefficient of the highly reflective load may be determined during the unterminating procedure. The TRL technique has become popular due to its unique advantage of not requiring that the standards be fully characterized. Hoer [4] has improved the accuracy of the TRL technique by increasing the number of standards.

In many cases the accuracy with which the fixture's S parameters are unterminated dominates the total measurement error. The OSL and TRL methods use greatly different solution algorithms. This complicates side-by-side evaluation and explains the absence of a direct comparison in the literature.

In this work a very general untermination algorithm is developed. The standards may be chosen arbitrarily. The OSL and TRL methods are shown to be special cases of this algorithm, making it easy to perform a direct comparison of the performance of the methods. The general algorithm developed in this work allows the use of any number of standards to increase accuracy.

In Section II an unterminating procedure based on a nonlinear least-squares algorithm is described. The OSL and TRL procedures are shown to be special cases of the more general unterminating procedure described here. A numerical technique for determining the accuracy of the unterminating procedure with different standard sets and under different assumptions is studied in Section IV. Comparison of the accuracy of the OSL and TRL procedures with each other and the more general unterminating procedure is easily accomplished by virtue of the OSL and TRL procedures being special cases of the more general procedure developed here. Procedures which achieve the greatest accuracy are recommended in Section V.

II. UTERMINATION PROCEDURE

Untermination is the process of determining the S parameters of an embedding network from measurements of various standards or known devices embedded in the network. To fix ideas, let the S parameters of the k th standard be S_{ij}^{ak} , let the S parameters describing the intervening fixture be S_{ij}^{f1} and S_{ij}^{f2} , and let the calculated S parameters of the fixture with the k th standard embedded inside of it be S_{ij}^{mk} , as shown in Fig. 1. For the special case in which all of the networks are reciprocal (i.e., having $S_{12} = S_{21}$), the measured S parameters S_{ij}^{mk} of the fixture with the k th standard embedded in it are related to the S

parameters of the k th standard S_{ij}^{ak} by

$$S_{ij}^{mk} = F_{ij}(S_{ln}^{f1}, S_{ln}^{f2}, S_{ln}^{ak}) \quad (1)$$

where the F_{ij} are complex nonlinear functions of the elements of the scattering matrices S_{ij}^{f1} , S_{ij}^{f2} , and S_{ij}^{ak} . The function F_{ij} may be found by calculating and cascading the transmission matrices of its parameters and then converting the resulting transmission matrix back into S parameters. In some cases, such as the TRL case, it is convenient to let the S parameters S_{ij}^{ak} of the standards be functions of the propagation constant γ of a transmission line and the reflection coefficient Γ_L of a terminating load.

If the S_{ij}^{mk} are defined as the S parameters measured by the analyzer of the fixture with the k th standard embedded in it, the S_{ij}^{mk} will differ from the S_{ij}^{mk} by some small measurement error. We can define a vector of residuals, r_{ij}^k , as

$$r_{ij}^k = S_{ij}^{mk} - S_{ij}^{mk} = S_{ij}^{mk} - F_{ij}(S_{ln}^{f1}, S_{ln}^{f2}, S_{ln}^{ak}). \quad (2)$$

The values \hat{S}_{ij}^{f1} and \hat{S}_{ij}^{f2} (and $\hat{\gamma}$ and $\hat{\Gamma}_L$ if they are not known) for which the sum of the squares of the residuals is a minimum are known as the least-squares estimates of S_{ij}^{f1} , S_{ij}^{f2} , γ , and Γ_L , respectively. Thus \hat{S}_{ij}^{f1} , \hat{S}_{ij}^{f2} , $\hat{\gamma}$, and $\hat{\Gamma}_L$ satisfy

$$\sum_{kij} |r_{ij}^k(\hat{S}_{ln}^{f1}, \hat{S}_{ln}^{f2}, \hat{\gamma}, \hat{\Gamma}_L)|^2 \text{ is a minimum.} \quad (3)$$

These estimates are, in the least-squares sense, the best approximation to the actual values of S_{ij}^{f1} , S_{ij}^{f2} , γ , and Γ_L for a given set of measurements S_{ij}^{mk} .

The least-squares estimates \hat{S}_{ij}^{f1} , \hat{S}_{ij}^{f2} , $\hat{\gamma}$, and $\hat{\Gamma}_L$ of S_{ij}^{f1} , S_{ij}^{f2} , γ , and Γ_L may be found by a variety of standard numerical techniques. In this work the IMSL subroutine DUNLSF,² which is based on the Levenberg-Marquardt algorithm and a finite difference Jacobian, was used [5]. The algorithm calculates the Jacobian of a real vector function for a set of guessed parameters using finite differences. The Jacobian is used to calculate better guesses for the parameters of a nonlinear function until convergence is reached.

The functions F_{ij} are complex vector functions of the complex parameters S_{ij}^{f1} , S_{ij}^{f2} , γ , and Γ_L . The subroutine DUNLSF was used to find the least-squares estimates of S_{ij}^{f1} , S_{ij}^{f2} , γ , and Γ_L . That is, the subroutine DUNLSF was used to find the values \hat{S}_{ij}^{f1} , \hat{S}_{ij}^{f2} , $\hat{\gamma}$, and $\hat{\Gamma}_L$ for which the sum of the squares of the residuals r_{ij}^k was minimized.

The subroutine DUNLSF was written to operate on real vector functions of real vectors. In order to find the solution to this problem, the real and imaginary elements of the complex vectors were mapped into real elements of real vectors for the purpose of solving the problem numerically. Thus for each possible value of k (one for each standard) and index ij (where i and j assume values of $(i=1, j=1)$, $(i=2, j=2)$, and $(i=2, j=1)$), an index l was assigned. The real and imaginary parts of each complex r_{ij}^k were mapped into indices $2l-1$ and $2l$ of a corresponding real vector G where

$$G = (\text{Re}(r_{11}^1), \text{Im}(r_{11}^1), \text{Re}(r_{22}^1), \text{Im}(r_{22}^1), \text{Re}(r_{21}^1), \text{Im}(r_{21}^1), \text{Re}(r_{11}^2), \text{Im}(r_{11}^2), \text{Re}(r_{22}^2), \text{Im}(r_{22}^2), \dots) \quad (4)$$

¹Under the assumption of reciprocity, the fixture transmission coefficients S_{21}^{fk} and S_{12}^{fk} are equal, resulting in only three unknown complex quantities per fixture two-port. After the fixture has been unterminated there remains an ambiguity in the sign of the fixture transmission coefficient. This ambiguity may be resolved by a transmission measurement.

²The IMSL subroutine library is a set of mathematical and statistical subroutines and is available from IMSL, 2500 Park West Tower One, 2500 City West Blvd., Houston TX 77042.

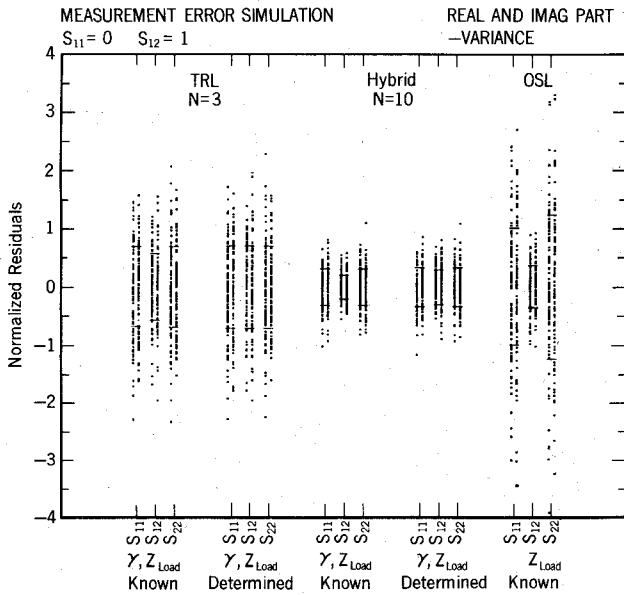


Fig. 3. A scatter plot of normalized residuals for an ideal fixture and several representative standard sets. The solid lines indicate the computed variance.

is the real vector function to be minimized in the least-squares sense. The arguments of G (the S_{ij}^f , S_{ij}^f , γ , and Γ_L) were arranged into a real vector in a similar way, allowing the subroutine DUNSLF to be used without modification.

III. ANALYSIS OF RANDOM ERRORS

Both random and systematic errors are introduced during the process of de-embedding. In most cases the systematic measurement error is due to systematic deviations in the standards from ideal standards and cannot be removed from the measurements. These systematic errors can only be estimated from fundamental considerations, and their treatment is beyond the scope of this work. Random measurement errors can be reduced by statistical analysis, and will be discussed here.

The variance-covariance matrix of the parameters of the F_{ij} is a measure of the error due to estimating those parameters from the data when the errors in the data are normally distributed. The variance-covariance matrix σ is approximately [6]

$$\sigma = s^2 (J^T J)^{-1} \quad (5)$$

where J^T is the transpose of J , the Jacobian of the real mapping of F , and s^2 , a scalar, is the variance of the data. The quantity s^2 depends on the accuracy of the measured data and is independent of the number of standards used. We shall call the quantity

$$e(S_{ij}^{fk}) = \sqrt{\sigma_{2121} / s^2} = \sqrt{(J^T J)^{-1}_{2121}} \quad (6)$$

the normalized variance of S_{ij}^{fk} , where l is the index which has been assigned to S_{ij}^{fk} in the solution process and k is 1 or 2. The normalized variance of S_{ij}^{fk} is a good measure of the accuracy of \hat{S}_{ij}^{fk} as determined by the algorithm. The normalized variances of γ and Γ_L are defined in an analogous fashion.

The suitability of the normalized variance for predicting errors was verified using a Monte Carlo technique. The S parameters at the plane of the analyzer were perturbed 100 times by adding small normally distributed random numbers to both their real and imaginary parts, yielding 100 sets of S_{ij}^{mk} . The least-squares estimates were calculated each time and the differences

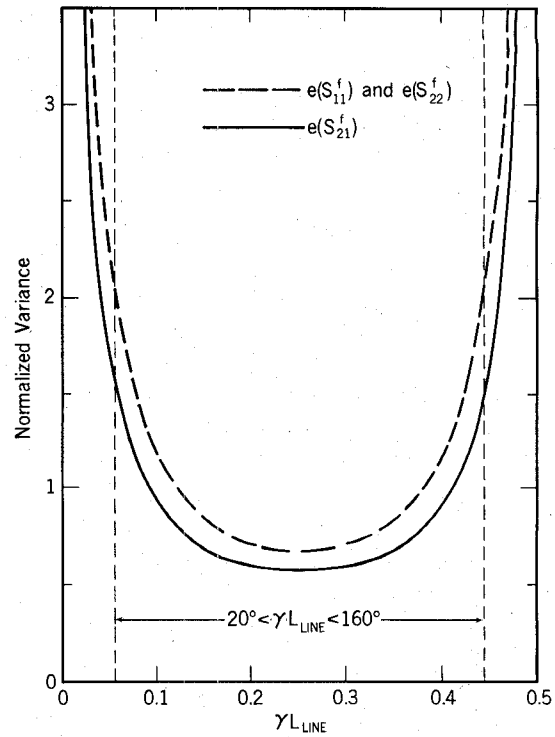


Fig. 4. The normalized variances of the TRL procedure are plotted as a function of the length of the transmission line standard.

between the real and imaginary parts of the residuals were plotted. The normalized variance of the least-squares estimates was also calculated and indicated on the plots.

This procedure was repeated for several sets of fixture S parameters and several sets of standards. In all cases, the computed normalized variance was found to predict the actual variance accurately. An example of such a plot is shown in Fig. 3, in which the fixture was assumed to have a transmission coefficient of 1 and a reflection coefficient of 0. In this case the number of least-squares estimates lying inside the calculated variance was 67.5%, extremely close to the ideal value of 68.3%, indicating that the calculated normalized variance was a good estimate of the accuracy of the algorithm.

IV. NUMERICAL RESULTS

In the special case in which the number of unknowns and the number of measurements are the same, there is a set of parameters of the F_{ij} for which the residuals are zero. This set of parameters of the F_{ij} are also the least-squares estimates of the F_{ij} . In both the OSL and TRL procedures the number of unknowns and the number of measurements are equal. Thus in both of these cases, the nonlinear least-squares algorithm used in this work converges to the same solution as the OSL and TRL procedures. This is useful because it allows the OSL and TRL procedures to be compared directly with the more general algorithm and with each other.

Most practical fixtures have low insertion losses and small reflection coefficients. Thus the normalized variances of an ideal symmetrical fixture with no transmission loss and a reflection coefficient of zero will be representative of the normalized variances of many practical fixtures. This ideal symmetrical fixture will be used as a benchmark for the purpose of comparison.

As an example of the utility of this benchmark, consider the TRL untermination procedure. If the delay line standard is a

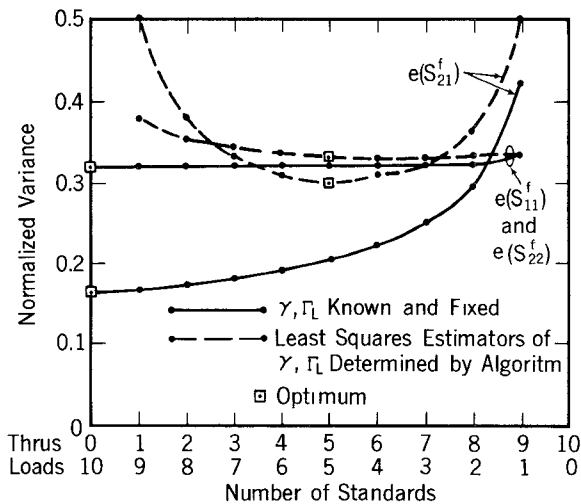


Fig. 5. The normalized variances of the untermination procedure are plotted as a function of the number of thru lines and reflective loads. The total number of standards is always 10.

quarter-wavelength long, the accuracy of the procedure is optimum. It has been recommended by one manufacturer of automatic network analyzers that the length of the line be greater than 20° and less than 160° . Fig. 4 shows the normalized variances of the TRL procedure calculated from (6) for an ideal symmetrical fixture. It is clear from the figure that the line length should indeed be constrained within this interval.

In Fig. 5 the normalized variances for an unterminating procedure utilizing n thru-line standards and m reflection standards are plotted. In each case the total number of standards was 10. The length l_i of the i th thru-line standard was chosen so that $\gamma l_i = \pi(i-1)/n$. The reflective standards were distributed uniformly around the outside of the Smith chart. Thus the i th reflective standard was composed of two transmission lines of length $\gamma l_i = \pi(i-1)/m$ terminated in open circuits, one connected to each port. These standards are depicted schematically in Fig. 2(c). (It might be thought that the use of zero-length standards restricts the applicability of this method. It is easy to show, however, that virtually identical results can be obtained by offsetting all of the standards by a set amount and later adjusting the reference plane of the measurement.)

The normalized variances are plotted for two cases. In the first case, the propagation constant γ and the reflection coefficient Γ_L of the open circuit are assumed known and are fixed during the untermination algorithm. In the second, the propagation constant γ and the reflection coefficient Γ_L are assumed to be unknown and their least-squares estimates are determined during the untermination procedure. In both cases the characteristic impedances of the thru lines and offset lines are assumed to be known and identical, and provide the characteristic impedance to which the system is calibrated.

It can be seen from Fig. 5 that if the propagation constant γ and the reflection coefficient Γ_L are known and fixed during the untermination procedure, the accuracy of the untermination procedure increases as more loads and fewer thru lines are used. When no reflective loads are used the normalized variance becomes infinite as no reference plane can be established.

The figure also shows that if the propagation constant γ and the reflection coefficient Γ_L are unknown and their least-squares estimates are found during the solution procedure, the accuracy of the untermination procedure is greatest when equal numbers of thru lines and reflective loads are used for standards. In this

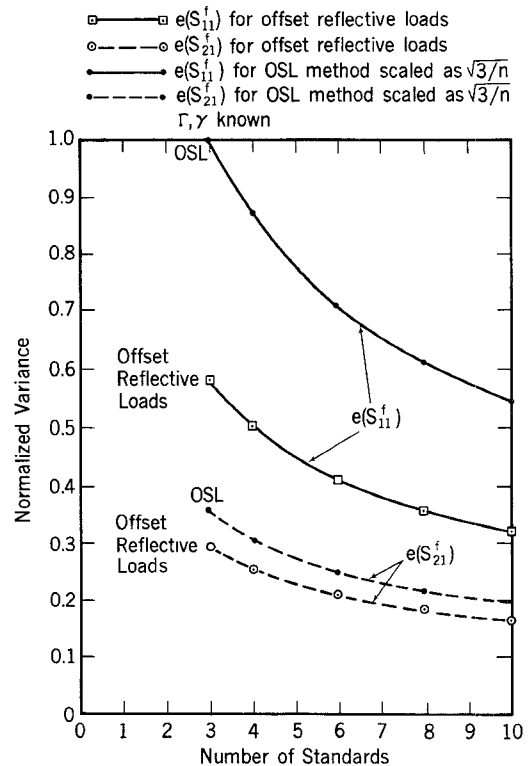


Fig. 6. The normalized variances of the untermination procedure are plotted as a function of the number of reflective loads. The propagation constant γ of the transmission lines in which the loads are offset and the reflection coefficient Γ_L of the offset loads are assumed to be known.

case the normalized variance is infinite when either no loads or no thru lines are used, again because a reference plane cannot be established.

In Fig. 6 the normalized variances of fixture S parameters are plotted as a function of the number of standards. In this case γ and Γ_L are assumed known. The standards were of the reflective type, with the i th standard consisting of two transmission lines of length $\gamma l_i = \pi(i-1)/n$ terminated in an open circuit. The improved accuracy as the number of standards is increased is clearly apparent in the figure.

Also plotted in the figure, for purposes of comparison, are the normalized variances of the OSL method scaled by $\sqrt{3}/n$, the factor by which the normalized variance would be decreased if redundant measurements were made. The figure shows clearly that the reflective standards provide greater accuracy in unterminating than the OSL method, at least for the case studied here.

In Fig. 7 the normalized variances are plotted for the case in which the least-squares estimates of γ and Γ_L are found during the untermination procedure. In this case, the S_{ij}^{ak} are functions of γ and Γ_L . This is also the case for the TRL method, and its normalized variances scaled as $\sqrt{3}/n$ are plotted in Fig. 7 for comparison. A comparison of Figs. 6 and 7 reveals that the TRL method is much closer to optimum than the OSL method.

V. CONCLUSION

A general unterminating technique has been described. The technique was used to investigate de-embedding under the assumptions that all measurement errors are random and normally distributed and that the standards are distributed uniformly around the Smith chart. It was shown that for any given number of standards, the greatest accuracy under these assump-

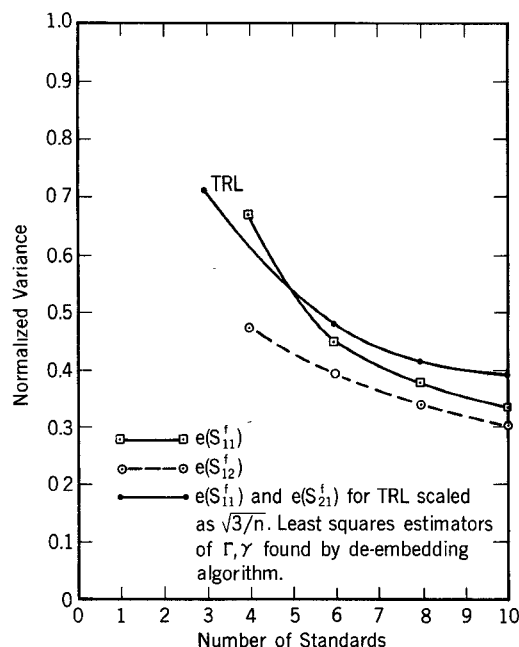


Fig. 7. The normalized variances of the untermination procedure are plotted as a function of the number of thru lines. The least-squares estimates of the propagation constant γ of the transmission lines and the reflection coefficient Γ_L of the offset loads are found during the untermination procedure.

tions is achieved by utilizing a large set of known reflective loads. When the propagation constant and the reflection coefficients of the standards are not known, then equal numbers of thru lines and reflective loads give the highest accuracy, although not as high as when the propagation constant and reflection coefficients are known. It was shown that the OSL technique was considerably less accurate than using sets of offset reflective loads.

The technique of unterminating illustrated here is very general and could be easily used to calibrate network analyzers. While the algorithm is computationally intensive, it may find application in practical situations because of its flexibility and accuracy. Furthermore, the variance of the data s^2 may be estimated from the residuals if more than three measurements are taken, allowing the variances of the least-squares estimates in the untermination procedure to be estimated. The variance of the de-embedded data could then be estimated as well.

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Microwave Radiometric Imaging at 3 GHz for the Exploration of Breast Tumors

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Abstract—A process of microwave radiometric imaging working at 3 GHz permits the mapping of radiometric intensities on a square area about half a decimeter on a side. These data, translated in terms of colored image, point out the existence of lateral temperature gradients in the tissues. This system was initially used in order to examine large breast tumors; at present, it is also used for the detection of smaller, impalpable tumors. We try to define the rules for the characterization of benignity or malignancy of small tumors which appear in a mammographic examination (X rays). The definition of an appropriate parameter, deduced from this image processing, seems to make it possible to indicate if the tumor is benign or malignant.

I. INTRODUCTION

For several years, because women are more aware of breast lumps, more breast tumors have been discovered at an early stage. Therefore, new problems arise for differential diagnosis. Indeed, these tumors are often so small that they cannot be investigated with clinical examination. Because some of these lumps cannot be characterized by mammography or percutaneous cytology, it is often necessary to resort to surgery, although most of the lumps are not cancers. This is quite unsatisfactory since the cost is not negligible, no surgical procedure is harmless, and scars may change the breast structure enough to prevent early diagnosis of any future cancer, not to mention aesthetic and psychological problems. Therefore, a noninvasive method effective for tumor discrimination would be very welcome. For several years, microwave radiometric imaging has been used in an attempt to increase the reliability of diagnosis [1]-[12].

We have conceived an imaging process using a multiprobe radiometer which increases the number of measurements and improves their localization. We explain how this process is used now for the examination of tumors of the breast, in order to try to define the rules for the characterization of benignity or malignancy, mainly for situations where the usual screening process—mammography—has failed.

II. MATERIALS AND METHOD

The multiprobe radiometer [8], [9], working in the bandwidth 2.5-3.5 GHz, is made of a classical low-noise receiver with a gain of 50 dB and a noise factor of 5.5 dB. Consequently, its sensitivity is $\pm 0.1^\circ\text{C}$. The multiprobe is a result of the juxtaposition of six open-ended apertures of rectangular waveguides

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